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Enhancing Students' Achievement in Descriptive Statistics by Emphasizing Statistical Reasoning

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Abstract

This study conducted classroom action research as a means to enhance students' achievement in descriptive statistics by emphasizing statistical reasoning. Statistical reasoning is the method people reason with statistical ideas and make sense of statistical information. In this research, it was categorized into six groups which are reasoning about: (1) data collection, (2) frequency distributions, (3) measures of central tendency, (4) measures of positions, (5) measures of dispersions, and (6) standard scores and normal distributions. The participants include 46 Grade 11 students in the first semester of academic year 2018 at a secondary school in Bangkok, Thailand. The topic in this study is descriptive statistics comprising: (1) statistics and data collection, (2) frequency distributions, (3) measures of central tendency, (4) measures of positions, (5) measures of dispersions and (6) standard scores and normal distributions. Instruments in this study consist of 8 lesson plans, achievement test, and reflection form. Teaching and learning lasted for 42 periods each at 50 minutes. This study had three cycles. Each cycle was composed of planning, acting, observing and reflecting. The results show that the effectiveness index is 0.60 which indicates that the students' achievement was enhanced to the level required by the researcher (accepted effectiveness index must be greater than or equal 0.50). From reflections, the researcher found that using statistical reasoning increases students' understanding and decreases students' misconceptions. Statistical misconceptions of students include: misleading graphs, computation of combined means, using inappropriate central tendency, interpretation and properties of central tendency and dispersions.

Introduction

Statistics is a science concerning (1) formulating a statistical question, (2) designing a plan for collecting useful data, (3) analyzing the data and (4) interpreting the results (Franklin et al., 2007). For years, the study of descriptive statistics in schools has been proposed by mathematics curriculum developers as a part of school mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) issued the Principles and Standards for School Mathematics to be used in school mathematics as guidelines in sequencing contents and in teaching from kindergarten to grade 12. The guidelines

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from NCTM were used as ways to adjust statistics curriculum in various countries including Thailand. Thailand's core curriculum prescribed statistics in strand 5 - data analysis and probability. The strand was composed of three learning standards. They specified that students could apply statistics and probability in: (1) analyzing data, (2) predicting an event and (3) making decision and solving problems (IPST, 2008). So, all the above learning standards were crucial in learning and teaching statistics.

In learning statistics, previous studies revealed that students had encountered difficulties in understanding some content in descriptive statistics even though it appeared in the core curriculum for a long time. Ismail & Chan (2015) identified in their study that students' misconceptions were about measures of central tendency; in addition, Zawojewski & Shaughnessy (2000) found that students could not choose appropriate measures of central tendency to best represent center of a data set. Moreover, the study from Ismail & Chan (2015) and from Callingham (1997) revealed that students perceived mean as a more appropriate measure of central tendency than that of median. Batanero, Tauber, & Sánchez (2004) studied about learning normal distributions. The study identified the difficulties in discrimination between theoretical models and empirical data. Some students were confused between the empirical data distribution and the theoretical distribution fitted to a data set such as not understanding what normal distributions had been used for.

Besides various studies, many books are written about statistical misconceptions and statistical reasoning. Some books present statistical misconceptions for the general public to avoid misunderstanding in real life situations. Some books present statistical misconception for students to avoid misunderstanding in descriptive statistics. Huff & Geis (1954) mentioned that statistics was easy to mislead if the readers had no experience and/or enough knowledge. Author wrote the book, How to Lie with Statistics, which identified many interesting points such as choosing bias sample, choosing inappropriate measures of central tendency, and using graphs which intentionally led the readers to misunderstand. In addition, Hung, Wah, Kwong, & Man (2010) wrote a chapter in a textbook about the use and misuse of statistics. It shows the misuse of diagrams, and misuse of average. In the book, Statistical Reasoning for Everyday Life, Bennett, Briggs, & Triola (2003) mentions the reasoning in sampling, data types, visual display of data, measures of central tendency, measures of variation, and normal distributions. Furthermore, the book also emphasizes statistical reasoning regarding how a reader should believe a statistical study and a few cautions of reading graphs.

In addition, some studies identified learning problems. Chance, Mas, & Garfield (2004)stated that many students are able to memorize formulas and definitions instead of reasoning with a problem. Students were often unable to understand the underlying processes or properly interpret the results from calculations. Another study also stated that students were not confident in their own reasoning abilities. Students believed that they should only use formulas and definitions in textbooks to answer questions correctly (Slauson, 2008).

Many mathematic educators have tried to solve these problems. There are many opinions to improve teaching and learning. Garfield & Franlin (2011) stated that learning should be focused on understanding statistical concepts and on statistical reasoning rather than only using the required computations and formulas. It is important that all students understand the basic of descriptive statistics through statistical reasoning.

For Thai students, the problems in learning are similar to the above stated problems. In pre-practicum course, the researcher interviewed students and found some problems which were obstacles for them to understand descriptive statistics and decrease their achievement. Some of the problems are as follows:

1. Some students could not explain why they had to arrange a data set into group or ungroup data.
2. Some students thought that mean was always the best answer for all data.
3. Some students did not even look for outliers.
4. Most students remembered only formulas without understanding how they came to the formulas.
5. Some students could not interpret the meaning of the number computed from formulas such as arithmetic means, standard deviations, and standard scores.

From the above problems, the researcher was interested in improving students' achievement by emphasizing statistical reasoning in teaching so that students learned descriptive statistics with understanding, perceived correct and meaningful concepts, decreased their misconceptions, and increased their achievement.

Objectives

The objective of this study is to enhance students' achievement in descriptive statistics by emphasizing statistical reasoning.

Research questions

1. Is applying statistical reasoning effective in enhancing students' achievement?

2. Does applying statistical reasoning increase students' cognitive behavior?

For research question 1, the researcher used the effectiveness index to indicate the enhancing of students' achievement. For research question 2, the researcher used the results from pre-test and post-test to indicate the increase of cognitive behaviors.

Literature review

The following section presents an overview of action research, reasoning, statistical reasoning, and students' achievement.

Action research

Action research was introduced by a German-American social psychologist known as Kurt Lewin. Action research has emerged as a popular tool for professional development, especially in the field of education (Grundy, 1995). It is explained as a self-reflective form of investigation performed by participants in social or educational situations to improve their practices or understanding of these practices (Carr & Kemmis, 1983; Kemmis & McTaggart, 1988). In terms of teacher professional development, Carr & Kemmis (1983) stated that it was beneficial for teachers to conduct research about their classroom practices and teaching skills.

There are several models from prior research in which they define the action research steps such as Hopkins, Kemmis and McTaggart, Elliot, and O'Leary but in this study the researcher used the model of Kemmis and McTaggart (Koshy, 2005). The essential characteristic cycle of this model is (1) plan, (2) act and observe and (3) reflect.

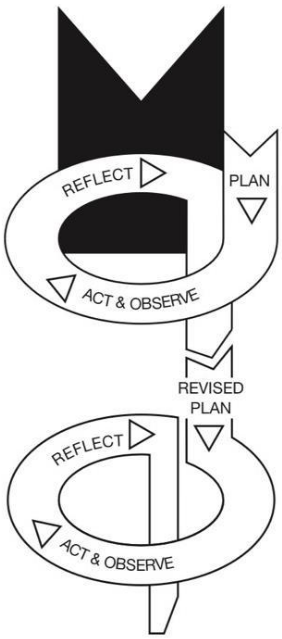

Figure 1 The Action Research Spiral (Kemmis & McTaggart, 1988)

Thinking and reasoning

Thinking is the ability to obtain a reasonable conclusion based on the provided information. It can lead to prediction based on situation of the problems. Giving reasoning includes explanation and the confirmation (Krulik & Rudnick, 1993). Krulik and Rudnick divided thinking into 4 levels which are recall, basic, critical, and creative (figure 2). Recall is considered close to a natural thinking skill that is almost automatic or reflexive. Basic includes the understanding and recognition of concepts as well as the application of these concepts in problems. Critical is thinking that examines, relates, and evaluates all aspects of a situation or a problem. Creative is thinking that is original and reflective and that produces a complex product. This level of thinking is an invention that is thought or imagined by oneself.

Reasoning is a part of thinking that lies beyond a recall level. It is divided into three levels: basic, critical, and creative (Krulik & Rudnick, 1993).

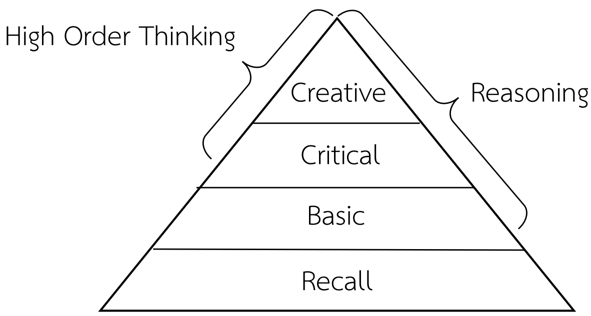

Figure 2 Hierarchy of Thinking (Krulik & Rudnick, 1993)

Statistical reasoning

Statistical reasoning is defined as the ways people reason with statistical ideas and make sense of statistical information (Garfield & Gal, 1999).

Previous studies have mentioned statistical reasoning. According to the study of Garfield (2002), they described statistical reasoning as the abilities of people are able to do with the statistic contents and the skills that they show in applying statistical concepts in problem solving. Statistical reasoning is presented as a system of three phases which are (1) understanding (seeing a specific issue as similar to a class of problem), (2) planning and performing (applying effective approaches to solve the problem) and (3) assessment and interpretation (interpreting the finding as it applies to the original issue) (Garfield, 2002).

In this study, the scope of statistical reasoning covers reasoning about: (1) data collection, (2) frequency distributions, (3) measures of central tendency, (4) measures of positions, (5) measures of dispersions and 6) standard scores and normal distributions.

Students' Achievement

Students' achievement is the level of students' success in learning descriptive statistics. It can be measured by using effectiveness index (E.I.). According to Kidrakan (2002), he defined effectiveness index as:

$$E.I.\ \frac{(P_1 - P_2)}{(Total - P_1)}\text{, where}$$

P_1 is the sum of all student's score from pre-test.
P_2 is the sum of all student's score from post-test.
Total is the product of the number of students and the full score of the test.

The effectiveness index is a ratio of differences. The maximum value of E.I. is 1.00 but it cannot be less than -1.00. Effectiveness index identifies the percentage of students' enhancement from the beginning (pre-test). Accepted effectiveness index of any innovation should be greater than or equal 0.5 (Kidrakan, 2002).

Conceptual framework

Generally, action research aims to solve problems in a specific setting or to constantly refine practice of practitioners. This classroom action research used the action research model presented by Kemmis and McTaggart (Kemmis & McTaggart, 1988).

Research methodology

1. Participants

The participants in this study are 46 Grade 11 students who studied in the first semester of academic

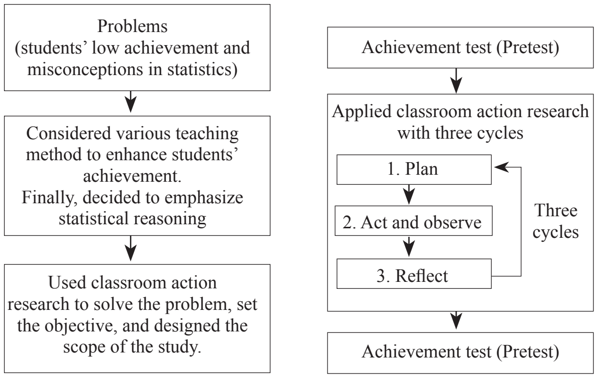

Figure 3 Conceptual Framework of this Study

year 2018 at a high school in Bangkok, Thailand.

2. Variables

2.1 Independent variable are teaching that emphasize statistical reasoning.

2.2 Dependent variable are students' achievement.

3. Research instruments

The instruments used in this study consist of lesson plans, achievement test, and reflections form.

3.1 Lesson plans

Lesson plans emphasize students' statistical reasoning to enhance students' achievement in descriptive statistics. There were 8 lesson plans for 42 periods with 50 minutes in each period.

3.1.1 The design of lesson plans

The designing of lesson plans are as follows:

1) The researcher studied the scope and sequence of lesson plans on descriptive statistics that integrated statistical reasoning. Mathematics content of Grade 11 descriptive statistics are as follows:

(1) Statistics and data collection
(2) Frequency distributions
(3) Measures of central tendency
(4) Measures of positions
(5) Measures of dispersions
(6) Standard scores and normal distributions

2) The researcher studied curriculum documents, textbooks and research about reasoning to design lesson plans. Some of the previous research and books were used as guidelines to teach statistics by emphasizing statistical reasoning. They were: (1) about memorizing all formulas instead of reasoning to understand the content (Chance, Mas, & Garfield, 2004), (2) about misconceptions in each type of measures of

central tendency (Ismail & Chan, 2015), (3) about choosing suitable measure of central tendency (Hung, Wah, Kwong, & Man, 2010) and (4) about types of data (Bennett, Briggs, & Triola, 2003).

3) The researcher set content, learning objectives, concepts, learning activities, materials and resources, desirable characteristics, measurement of learning, worksheets and assignment. Then, the researcher designed lesson plans and learning activities emphasizing reasoning to enhance students' achievement.

4) The researcher designed lesson plans for teaching each topic and then the lesson plans were revised by advisor.

5) The researcher adjusted each lesson to follow new situations that were found from reflection in each cycle to enhance students' achievement (repeat step 4 again).

3.1.2 Teaching step in each period

Each lesson plan emphasized statistical reasoning. Teaching steps of each period are shown as follows:

1) Reviewing previous knowledge: The researcher reviewed previous knowledge to prepare students. In addition, the researcher discussed and explained the students' homework in which students made mistakes and had misinterpretation. Then, the researcher identified learning objectives to students.

2) Guided questions: The researcher gave students questions that were related to real life situations as an introduction of the content.

3) Teaching phase: The researcher presented current knowledge through activities and examples.

4) Guided practices: The researcher gave questions or problems to students through activity sheets.

5) Independent practice: Each student tried to think for themselves and then gave the solutions and exchanged ideas with their partners including instructor. The researcher evaluated the environment in the classroom and provided feedback for improvements.

6) Closure: The research summarized the lesson for better understanding of students and assessed students' performance by observing students' behaviors from activities, answers, and reasoning to support their answers.

3.2 Achievement test

The achievement test was used to assess students' achievement. The test consisted of 20 multiple-choice items, and 4 written items. The total score is 36 points. The achievement test was used as both pre-test and post-test.

3.2.1 The method designing achievement Test

The designing of achievement test is as follows:

1) The researcher searched for misconceptions and errors in statistics from experts or research. Prior studies mentioned in the introduction were used as guidelines for researcher to construct the achievement test which are: (1) choosing appropriate measure of central tendency from Ismail and Chan study, (2) differencing between theoretical models and empirical data in normal distributions from Batanero, Tauber, and Sánchez study and (3) meaning of formulas in descriptive statistics from Chance, Mas, and Garfield.

2) The researcher analyzed the institutional curriculum, learning concepts, learning standards, learning indices, and learning objectives of the course.

3) The researcher constructed the achievement test by designing a table of specification that related to statistics content and students' behaviors. This test was used for both pre-test and post-test.

4) The constructed test was submitted to advisor for checking and revising. At first the researcher created thirty-five items in the achievement test. After revising, the achievement test was composed of twenty multiple-choice items and four written items.

5) The researcher presented the revised test to three expert teachers for checking the content validity. The content validity was evaluated by using the Index of Item Objective Congruence (IOC). In this research, each test item with IOC between 0.60 to 1.00 was chosen. Index of Item Objective Congruence of each item computed from the following formula

$$\text{IOC} = \frac{\sum R}{N}, \text{ where}$$

$\sum R$ means sum of scores checked by three experts.

N means the number of experts.

The rating of each item will be +1, 0, or -1.

+1 means the test item was congruent with the objectives.

0 means not sure the test item was congruent with the objectives.

-1 means the test item was not congruent with the objectives.

6) The researcher adjusted the test as recommended by the three experts.

7) The researcher checked each test item, tried them out with students in another class, and then revised the test again.

3.2.2 Classification of content, levels of behaviors and the number of test items classification of content, levels of behaviors and the number of test items are shown in Table 1.

Table 1 Table of specifications of test items for descriptive statistics

Contents	Number of items in each level of behaviors			
	Recall	Comprehension	Application	Analysis
1. Statistics and data collection	1	1		
2. Frequency distributions	1		3	
3. Measures of central tendency		1	4	3
4. Measures of positions			2	
5. Measures of dispersions		1	2	2
6. Standard scores and normal distributions			3	
Total (percent)	**2 (8.3 %)**	**3 (12.5 %)**	**14 (58.3 %)**	**5 (20.8 %)**

3.3 Reflection form

The reflection form aimed to reflect students' behaviors in the classroom in order to improve the plans and the students' understanding. They are divided into four parts which are students' knowledge, students' behavior, students' assignment, and improvement plan.

Students' knowledge consisted of collecting the information about the previous knowledge of the students. The researcher prepared the questions that related to the lesson plan on each period. Moreover, some questions required discussion among students to correct their understanding. This was conducted at the beginning of each period. Students' behaviors were observed and recorded in each period. Students' assignments were checked from the students' homework and students' workbook. This part was looking for the mistake from students' assignments. Improvement plans were about the ways to develop students based on the problems found from students' knowledge, students' behaviors and students' assignments.

4. Data collection

The data collection of this research was performed during the full first semester. It took 42 periods with 50 minutes in each period. The researcher collected data from pre-test, post-test and teacher's reflections as follows.

1) All students took a pre-test before learning descriptive statistics. The test lasted one period.

2) The researcher taught the class for forty-two periods. At the end of each topic, the researcher assigned a worksheet and homework to students. Then, the researcher recorded first reflection at the third lesson plan. The researcher did the same for two more cycles at the fifth and the sixth lesson plans, respectively.

3) After the last topic, all participants were asked to take the post-test.

5. Data analysis

1. Quantitative data: The data were collected from pre-test and post-test and were analyzed by using effectiveness index.

2. Qualitative data: The researcher analyzed qualitative data from teachers' reflections which were: 1) students' knowledge, 2) students' behaviors, 3) students' assignments and 4) improvement plan.

Results

1. Results from the tests

This study aimed to enhance students' achievement by emphasizing statistical reasoning. Quantitative data was analyzed to determine students' achievement. Table 2 shows the scores from pre-test, post-test, and effectiveness index.

Table 2 Effectiveness index of students' ahievement

Test	Mean	N	Std. deviation	Sum of students' score	Effectiveness index
Pre-test	11.80	46	3.83	543	
Post-test	26.33	46	4.02	1,211	E.I. = 0.60
Total score				1,656	

Table 2 presents the value of effectiveness index calculated from pre-test score, post-test score and total score. The sum of 46 students' scores on pre-test, post-test, and total are 543, 1,211, and 1,656, respectively. The value of the effectiveness index is 0.60 which is greater than 0.50. So, it could be concluded that students' achievement was enhanced as required by research question one.

In addition, the researcher analyzed the students' pre-test and post-test scores by calculating the mean score and standard deviation to describe the central tendency and dispersion of students' scores. The data showed the means for pre-test and post-test, the number of the students, and the standard deviations. The mean of the post-test is much higher than that of the pre-test; however, the pre-test standard deviation is slightly lower than that of the post-test.

Students' content scores across behaviors

Table 3 and 4 shows students' content scores across behaviors for both pre-test and post-test.

Table 3 Students' content scores across levels of behavior for pre-test

Contents	Average pre-test scores			
	Recall $\bar{x}$ (total)	**Comprehension $\bar{x}$ (total)**	**Application $\bar{x}$ (total)**	**Analysis $\bar{x}$ (total)**
1. Statistics and data collection	0.20 (1)	0.93 (1)		
2. Frequency distributions	0.83 (1)		1.91 (6)	
3. Measures of central tendency		0.50 (1)	2.60 (7)	1.14 (6)
4. Measures of positions			0.59 (2)	
5. Measures of dispersions		0.89 (1)	0.28 (5)	0.83 (2)
6. Standard scores and normal distributions			1.11 (3)	

Table 3 reveals that in the pre-test, students' content scores across levels of behavior are lower than that of the half of the total score in nearly every cell and so are for the overall $\bar{x}$, especially, with application and analysis.

Table 4 Students' content scores across levels of behavior for posttest

Contents	Average post-test scores			
	Recall $\bar{x}$ (total)	**Comprehension $\bar{x}$ (total)**	**Application $\bar{x}$ (total)**	**Analysis $\bar{x}$ (total)**
1. Statistics and data collection	0.26 (1)	0.98 (1)		
2. Frequency distributions	1.00 (1)		5.43 (6)	
3. Measures of central tendency		0.76 (1)	6.13 (7)	3.61 (6)
4. Measures of position			1.46 (2)	
5. Measures of dispersions		0.89 (1)	2.50 (5)	1.04 (2)
6. Standard scores and normal distributions			2.26 (3)	

Table 4 reveals that in post-test students' content scores across behaviors are higher than that of the half of the total score in nearly every cell and so are for the overall $\bar{x}$.

Table 5 and Table 6, shows students' scores in each content and in each behavior for pre-test and post-test.

Table 5 Summary of average scores in each level of behaviors for pre-test and post-test

Contents	Summary of average scores across behavior's level			
	Recall $\bar{x}$ (total)	**Comprehension $\bar{x}$ (total)**	**Application $\bar{x}$ (total)**	**Analysis $\bar{x}$ (total)**
Pretest	1.02 (2)	2.33 (3)	6.49 (23)	1.97 (8)
Posttest	1.26 (2)	2.63 (3)	17.78 (23)	4.65 (8)

5 demonstrates that the average scores across behavior's level in post-test is much higher than that of the half of the total scores while the average pre-test scores across behavior's level is lower than that of the total scores. So, the applying stati`stical reasoning could increase students' cognitive behavior.

Table 6 Summary of average content scores for pre-test and post-test

Contents	Summary of average scores across contents	
	pre-test $\bar{x}$ (total)	**post-test $\bar{x}$ (total)**
1. Statistics and data collection	1.13 (2)	1.24 (2)
2. Frequency distributions	2.74 (7)	6.43 (7)
3. Measures of central tendency	4.24 (14)	10.50 (14)
4. Measure of position	0.59 (2)	1.46 (2)
5. Measures of dispersions	2.00 (8)	4.43 (8)
6. Standard scores and normal distributions	1.11 (3)	2.26 (3)
Overall $\bar{x}$ (total)	**11.80 (36)**	**26.33 (36)**

Table 6 shows that the overall x of pre-test scores are lower than half of the total scores in each content except content one. The overall x of post-test scores are nearly the total scores in each content except content one and five.

In conclusion, the results from all above tables shows that students' average scores in content across behaviors levels for post-test are higher than that of pre-test scores. It could be summarized that the students' achievement is enhanced due to the effect of emphasizing statistical reasoning.

2. Results from reflections

Reflection is a part of the classroom action research cycle. In this study, the researcher applied three cycles that consisted of students' knowledge, students' behavior, students' assignment, and improvement plan. From each reflection, the researcher also looked for misconceptions, reasoning, and students' understanding.

Reflection from cycle 1

1) Students' knowledge:

Students could not identify whether a data set is quantitative or qualitative, and they did not understand why they had to construct a frequency table.

2) Students' behavior:

Some students had trouble in paying attention on learning because they thought that they had studied statistics before.

3) Students' assignment:

The students' worksheet shows their confusion between qualitative and quantitative data. They thought that citizen ID number, ages, and salaries were quantitative data. In representation of data, students showed confusion on using histogram or bar chart. In addition, using different scales in drawing graphs or histograms of a data set resulted in different shapes of graph which led to different understanding.

4) Improvement plan:

For the students who did not pay attention in class because they had studied statistics before, the researcher assigned advanced problems for them. Moreover, the researcher planned to enhance statistical reasoning about types of data by giving more examples together with discussion and made a judgment whether a data set were qualitative or quantitative. For students' confusion between histograms and bar charts, the key points discussed using bar charts for qualitative data and using histograms for continuous quantitative data.

Reflection from cycle 2

1) Students' knowledge:

After the researcher used examples to explain and to discuss with students about types of data, some students could identify the difference between qualitative and quantitative data.

For statistical formulas in measures of central tendency and measures of positions, most students could remember the formulas and plug in the data without understanding the meaning of the variables in the formulas. Therefore, they easily forget those formulas and had difficult to interpret the results.

Some students could not justify the use of mean, median, or mode to tell the center of a data set, and some students became confused between positions of data and numerical values of that position.

2) Students' behavior:

Some students did not pay attention to the lessons and they missed some important key points. So, the researcher had to explain again.

3) Students' assignment:

From some students' homework, students could not (1) choose the appropriate central tendency, (2) differentiate between positions of data and numerical values of the positions and (3) explain the differences between the formulas of grouped and ungrouped data.

From observation, misconceptions found during the class includes: (1) computing the combined means, (2) median positions, (3) mode conceptions, (4) confusion in using central tendency, (5) using symbols having the same pronunciation (σ and $\sum$) and (6) errors in using summation properties.

4) Improvement plan:

In the next classes, the researcher gave students data sets with outliers and asked them to compute mean, median, and mode. After the computation a discussion was held to see which obtained values showed the better central value. Then, teacher and students summarized together to obtain the reasoning about central value with the outliers in data sets. For the students who paid less attention, the researcher gave more problems to solve.

Reflection from cycle 3

1) Students' knowledge:

After students discussion regarding the comparison of each central tendency, students had better ideas in choosing appropriate central values. In addition, the researcher emphasized meaning to prevent mistakes between data values and their position values.

In teaching measures of dispersions, it was found that some students were confused in (1) interpreting meaning of some dispersions especially the mean0ing of standard deviation, (2) identifying formulas for grouped or ungrouped data and (3) identifying formulas for sample standard deviation or population standard deviation.

2) Students' behavior:

When the researcher asked students to do assignment about dispersions, some students complained that the computation was very complicated. So, some students did not pay attention to do the problems during the class. Therefore, the researcher allowed students to use a calculator.

3) Students' assignment:

From students' worksheet, the researcher found that students avoided or misunderstood to interpret the meaning of standard deviations.

From the researcher's observation, misconceptions found during the class activities were about the properties of standard deviation.

4) Improvement plan:

The researcher decided to add exercises about meaning and interpretation of each measure of dispersion. For the next lesson, the researcher emphasized statistical reasoning regarding why we had to use standard scores in computing area under normal curves.

In conclusion from the three cycles above, some students showed deficiencies in reasoning about (1) types of data, (2) representations of data, (3) statistical measures of central tendency and dispersion, (4) interpretation of the results, and (5) variables in formulas. In addition, some misconceptions in descriptive statistics were found about (1) computing the combined means, (2) confusion in using central tendency, (3) differentiating between positions of data and values of data, and (4) the properties of standard deviation.

Discussion

The students' achievement in descriptive statistics was enhanced after the researcher emphasized the statistical reasoning. Reasoning supported students in various ways such as understanding the concepts being studied, interpreting the results obtained, comparing the procedure used, justification of the solutions, and correcting errors.

For example, in finding the central value of a population with many outliers, using average will not give the true central value when compared to using median. Implementing reasoning helps students to not accept mode from a data set with bimodal or trimodal. In studying about normal distributions, reasoning based on normal distributions helps students understand why it is necessary to change any score (x) to standardized score (z) to find the area under a normal curve. In computing combined means of two samples, error expressed by some students was $\frac{(\bar{x}_1 + \bar{x}_2)}{2}$ instead of $\frac{(n_1\bar{x}_1 + n_2\bar{x}_2)}{(n_1 + n_2)}$. Using real data sets could help students' in comparing the two methods used. Interpretations of numbers computed from problems also supported students' understanding and increases reasoning ability.

The result of this study is in accordance with prior studies. Ganesan & Leong (2018) conducted a study on the effectiveness of Fathom instruction base to enhance statistical reasoning. A quasi-experimental non-equivalent pre-test/post-test study was conducted with a total of seventy-two form four students randomly assigned to a control group (n = 38) and an experimental group (n = 34). The experimental group was taught using dynamic software, Fathom; meanwhile the control group was taught without involving Fathom. The findings indicate that there is a difference in the mean score of post-test between experimental and control groups. The post-test scores showed that students who learned statistical reasoning using Fathom outperformed those in the control group.

Dijke-Droogers, Drijvers & Tolboom (2018) explored ways to enhance Grade 8 students' statistical literacy through within-class differentiation. The developed course materials consisted of a differentiated module in the Digital Mathematics Environment (DME), combined with investigation activities during classroom sessions. The material focused on statistical reasoning using visual representations made with TinkerPlots. It was concluded that this teaching arrangement indeed increased students' statistical literacy.

The research above applied Fathom and module DME in encouraging students' reasoning while in this study the researcher used course materials, problems, discussion, and questions. Kilpatrick, Swafford, & Findell (2001) offer teaching suggestions to encourage reasoning.

Suggestions

1. During teaching, teachers should prepare a short period of time for discussion or for group activities from well-prepared worksheet. From answering questions, discussions and group activities, students would summarize, compare, and contrast methods used in each statistics topic. Students could learn from their own thinking and reasoning and at the same time they learned and followed reasoning from their peers. These activities would increase students' understanding.

2. Additional and various real-life data from newspapers or reports such as polls or articles should be included for the class to face various reasoning from diverse situations. This can enhance students' statistical reasoning abilities.

3. For preservice teachers, learning activities from this study could help in preventing students from statistical misconceptions and to give ideas or situations to be integrated in the worksheet.

4. Using technology is necessary in 21st century. Ben-Zvi & Garfield, (2006) in the book, The challenge of developing statistical literacy, reasoning, and thinking, mentioned that using technology-assisted learning can support students' construction of statistical reasoning idea. Nowadays, there are various software programs for a teacher to use in the classroom. The popular software programs are, for example, BrightStat, Graphmatica, SPSS and SAS. BrightStat is a spreadsheet and data handling software for numerical computing and graphing. Graphmatica is a graphing program for easy plotting. SPSS and SAS are software programs for analyzing data. In this study, the researcher also used Excel to help to design worksheets. Sometimes the exercises in the books are not easy to calculate by oneself so the researcher brought Excel to help each student in computing. It is interesting to use excel or other software programs for

descriptive statistics in a secondary school to save time in calculating, drawing graphs or charts and to increase time for reasoning, experimentation, and discussion so that students could increase their achievement. For example, a textbook on business statistics (Aczel & Sounderpandian, 2006) includes in Chapter 1 descriptive statistics the use if Excel templates to compute various measures and to create charts.

5. Misconceptions were found during teaching and also from students' worksheets. They could be extended for one who is interested in conducting research in this area. Further research can look for more misconceptions in descriptive statistics to design the misconception test and for designing instruction plans to enhance students' achievement.

6. Set-up of problem-based learning was satisfied with learners to promote skills in communication and to use via technology (Piromnok, 2016). This can be applied in descriptive statistics teaching to enhance statistical reasoning.

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